ON THE TWIST–2 CONTRIBUTIONS TO POLARIZED STRUCTURE FUNCTIONS *

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ABSTRACT

The twist–2 contributions to the polarized structure functions in deep inelastic lepton–hadron scattering are calculated including the exchange of weak bosons and using both the operator product expansion and the covariant parton model. A new relation between two structure functions leading to a sequence of new sum rules is found. The light quark mass corrections to the structure functions are derived in lowest order QCD.

1. Introduction

The study of polarized deep inelastic scattering off polarized targets has revealed a rich structure of phenomena during the last years [1]. So far only the case of deep inelastic photon scattering has been studied experimentally. Future polarized proton options at high energy colliders as RHIC and HERA would allow to probe the spin structure of nucleons at much higher Q^2 (cf. ref. [2]) also. In this range Z–exchange contributions become relevant and one may investigate charged current scattering as well. For this general case the scattering cross section is determined by (up to) five polarized structure functions per current combination.

In the present paper we derive the relations for the complete set of the polarized structure functions including weak interactions in lowest order QCD which are not associated with terms in the scattering cross section vanishing as $m_{lepton} \to 0$. The calculation is performed applying two different techniques: the operator product expansion and the covariant parton model [3]. In the latter approach furthermore also the quark mass corrections are obtained.

As it turns out the twist–2 contributions for only two out of the five polarized structure functions, corresponding to the respective current combinations, are linearly independent. Therefore three linear operators have to exist which determine the remaining three structure functions over a basis of two in lowest order QCD. Two of them are given by the Wandzura–Wilczek[4] relation and a relation by Dicus [5]. A third *new* relation is found [6].

We construct the hadronic tensor using both Lorentz and time reversal invariance and current conservation. It is given by:

$$W_{\mu\nu}^{ab} = \frac{1}{4\pi} \int d^4x e^{iqx} \langle pS \mid [J_\mu^a(x), J_\nu^b(0)] \mid pS \rangle, \tag{1}$$

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where in framework of the quark model the currents are

$$J^a_\mu(x) = \sum_{f,f'} U_{ff'} \overline{q}_{f'}(x)\gamma_\mu(g^a_V + g^a_A\gamma_5)q_f(x). \tag{2}$$

In terms of structure functions the hadronic tensor reads:

$$\begin{aligned} W^{ab}_{\mu\nu} &= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})F^i_1(x,Q^2) + \frac{\widehat{p_\mu}\widehat{p_\nu}}{p.q}F^i_2(x,Q^2) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q_\lambda p_\sigma}{2p.q}F^i_3(x,Q^2) \\ &+ i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{p.q}g^i_1(x,Q^2) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p.qS^\sigma - S.qp^\sigma)}{(p.q)^2}g^i_2(x,Q^2) \\ &+ \left[\frac{\widehat{p_\mu}\widehat{S_\nu} + \widehat{S_\mu}\widehat{p_\nu}}{2} - S.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{(p.q)}\right]\frac{g^i_3(x,Q^2)}{p.q} \\ &+ S.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{(p.q)^2}g^i_4(x,Q^2) + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{(S.q)}{p.q}g^i_5(x,Q^2), \end{aligned} \tag{3}$$

with $ab \equiv i$ and

$$\widehat{p_\mu} = p_\mu - \frac{p.q}{q^2}q_\mu, \qquad \widehat{S_\mu} = S_\mu - \frac{S.q}{q^2}q_\mu. \tag{4}$$

Here $x = Q^2/2p.q \equiv Q^2/2M\nu$ and $Q^2 = -q^2$ is the transfered four momentum squared. p and S denote the four vectors of the nucleon momentum and spin, respectively, with $S^2 = -M^2$ and, $S.p = 0$. g_{V_i} and g_{A_i} are the vector and axialvector couplings of the bosons exchanged in the respective subprocesses. Different other assignements of structure functions were used by other authors (cf. [6,7] for a corresponding survey). They can be obtained as linear combinations of those defined in eq. (3).

2. Operator Product Expansion

We consider the spin–dependent part of the forward Compton amplitude, $T^{ij,spin}_{\mu\nu}$. It is related to the corresponding part of the hadronic tensor by

$$W^{ij,spin}_{\mu\nu} = \frac{1}{2\pi} Im T^{ij,spin}_{\mu\nu}, \tag{5}$$

where

$$T^{ij,spin}_{\mu\nu} = i\int d^4x e^{iqx}\langle pS \mid (T {J^i_\mu}^\dagger(x) J^j_\nu(0)) \mid pS\rangle\Big|_{spin}. \tag{6}$$

The forward Compton amplitude is expanded into local operators near the light cone and the operator expectation values are calculated. They are related to the moments of the structure functions introduced in eq. (3). In the present paper we are considering only the operators of twist 2. A complete account of the contributions of the operators of both twist 2 and 3 is given in ref. [7]. The following relations are obtained:

$$\int_0^1 dx x^n g^j_1(x,Q^2) \quad = \quad \frac{1}{4}\sum_q \alpha^q_j a^q_n, \qquad n = 0, 2..., \tag{7}$$

$$\int_0^1 dx x^n g_2^j(x,Q^2) = -\frac{1}{4}\sum_q \alpha_j^q \frac{n a_n^q}{n+1}, \qquad n=2,4..., \tag{8}$$

$$\int_0^1 dx x^n g_3^j(x,Q^2) = \sum_q \beta_j^q \frac{a_{n+1}^q}{n+2}, \qquad n=0,2..., \tag{9}$$

$$\int_0^1 dx x^n g_4^j(x,Q^2) = \frac{1}{2}\sum_q \beta_j^q a_{n+1}^q, \qquad n=2,4..., \tag{10}$$

$$\int_0^1 dx x^n g_5^j(x,Q^2) = \frac{1}{4}\sum_q \beta_j^q a_n^q, \qquad n=1,3... \ . \tag{11}$$

Here a_n^q is related to the expectation value $\langle pS|\Theta_S^{\beta\{\mu_1...\mu_n\}}|pS\rangle$ (see refs. [6,7] for a detailed description). The factors α_j^q and β_j^q are given by

$$\left(\alpha^q_{|\gamma|^2}, \alpha^q_{|\gamma Z|}, \alpha^q_{|Z|^2}\right) = \left[e_q^2, 2e_q g_V^q, (g_V^q)^2 + (g_A^q)^2\right] \tag{12}$$

$$\left(\beta^q_{|\gamma Z|}, \beta^q_{|Z|^2}\right) = \left[2e_q g_V^q, 2g_V^q g_A^q\right]. \tag{13}$$

Analogous relations to (7–11) are derived for the charged current structure functions (cf. ref. [7]). By analytic continuation of the moment index n into the complex plane one obtains the following relations between the twist–2 contributions to the structure functions $g_k^j|_{k=1}^5$:

$$g_2^i(x) = -g_1^i(x) + \int_x^1 \frac{dy}{y} g_1^i(x), \tag{14}$$

$$g_4^j(x) = 2x g_5^j(x), \tag{15}$$

$$g_3^j(x) = 4x \int_x^1 \frac{dy}{y} g_5^j(y), \tag{16}$$

where $i = \gamma, \gamma Z, Z, W$ and $j = \gamma Z, Z, W$. Eqs. (14) and (15) are the Wandzura–Wilczek [4] and Dicus [5] relations, and eq. (16) is a *new* relation which was firstly derived in ref. [6].

3. Covariant Parton Model

In the covariant parton model the hadronic tensor for deep inelastic scattering is given by

$$W_{\mu\nu,ab}(q,p,S) = \sum_{\lambda,i}\int d^4k f_\lambda^{q_i}(p,k,S) w^{q_i}_{\mu\nu,ab,\lambda}(k,q)\delta[(k+q)^2 - m^2]. \tag{17}$$

Here $w^{q_i}_{\mu\nu,ab,\lambda}(k,q)$ denotes the hadronic tensor at the quark level [6], $f_\lambda^{q_i}(p,k,S)$ describes the quark and antiquark distributions of the hadron, λ is the quark helicity, k the virtuality of the initial state parton,

$$k = xp + \frac{k^2 + k_\perp^2 - x^2M^2}{2x\nu}(q + xp) + k_\perp, \tag{18}$$

and m is the quark mass.

In the Bjorken limit $Q^2, \nu \to \infty$, $x = const.$ one obtains the following representation for the polarized neutral current structure functions including light quark mass effects:

$$
\begin{aligned}
g_1^j(x,\rho) &= \frac{\pi M^2 x}{8} \sum_q \alpha_q^j \int_{x+\frac{\rho}{x}}^{1+\rho} dy \left[x(2x-y)+2\rho\right] \tilde{h}_q(y,\rho), && (19)\\
g_2^j(x,\rho) &= \frac{\pi M^2}{8} \sum_q \alpha_q^j \int_{x+\frac{\rho}{x}}^{1+\rho} dy \left[x(2y-3x)-\rho\right] \tilde{h}_q(y,\rho) \\
&- \frac{\pi m^2}{4} \sum_q \gamma_q^j \int_{x+\frac{\rho}{x}}^{1+\rho} dy \tilde{h}_q(y,\rho), && (20)\\
g_3^j(x,\rho) &= \frac{\pi M^2 x^2}{2} \sum_q \beta_q^j \int_{x+\frac{\rho}{x}}^{1+\rho} dy (y-x) \tilde{h}_q(y,\rho), && (21)\\
g_4^j(x,\rho) &= 2x g_5(x), && (22)\\
g_5^j(x,\rho) &= \frac{\pi M^2}{8} \sum_q \beta_q^j \int_{x+\frac{\rho}{x}}^{1+\rho} dy \left[x(2x-y)+2\rho\right] \tilde{h}_q(y,\rho), && (23)
\end{aligned}
$$

with $\rho = m^2/M^2$, $\gamma_q^j = g_{A_a}^q g_{A_b}^q, j \equiv ab$, $y = x + k_\perp^2/(xM^2)$, and

$$
\tilde{h}_q(y,\rho) = \int dk^2 \hat{f}_q(y,k^2,\rho). \tag{24}
$$

The corresponding relations for charged current scattering are given in [7]. The expressions for g_1^{em} and g_2^{em} have been obtained in [9,8] already.

In the limit $\rho \to 0$ the three relations eqs. (14–16) between the different structure functions being derived previously using the operator product expansion are obtained choosing g_1 and g_5 as the two basis structure functions in lowest order QCD. On the basis of the above relations a series of sum rules can be derived. For $\rho \neq 0$ a new contribution to g_2 emerges for $g_{A_a} g_{A_b} \neq 0$ leading to a corresponding violation of the Burkhardt–Cottingham sum rule [10]. The first moment of the structure functions g_3 and g_4 is predicted to be equal for all current combinations also for $\rho \neq 0$ and

$$
\int_0^1 dx\, x \left[g_1^k(x) + 2 g_2^k(x)\right] = 0 \tag{25}
$$

holds for arbitrary quark masses in the case of charged current interactions. Also the Dicus relation [5] remains valid. A detailed discussion is contained in refs. [6,7] where also numerical results are presented.

4. Conclusions

We have derived the twist–2 contributions to the polarized structure functions in lowest order QCD including weak currents. The results obtained using the operator product expansion and the covariant parton model agree. In lowest order two out of five structure functions are independent for the respective current combinations and the remaining structure functions are related by *three* linear operators. A new relation between the structure

functions g_3^j and g_5^j was derived. As a consequence the first moment of g_3^j and g_4^j are predicted to be equal.

The light quark mass corrections to the structure functions $\left. g_k^j \right|_{k=1}^{5}$ were calculated in the covariant parton model. The first moments of the structure functions g_3 and g_4 are equal also in the presence of the quark mass corrections. The Dicus relation remains to be valid. The Burkhardt–Cottingham sum rule is broken by a term $\propto g_{A_a} g_{A_b} m^2/M^2$, i.e. for pure Z exchange and in charged current interactions.

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